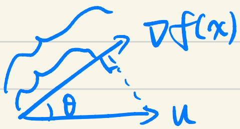
Directional derivative

$\nabla f(x) \cdot u$

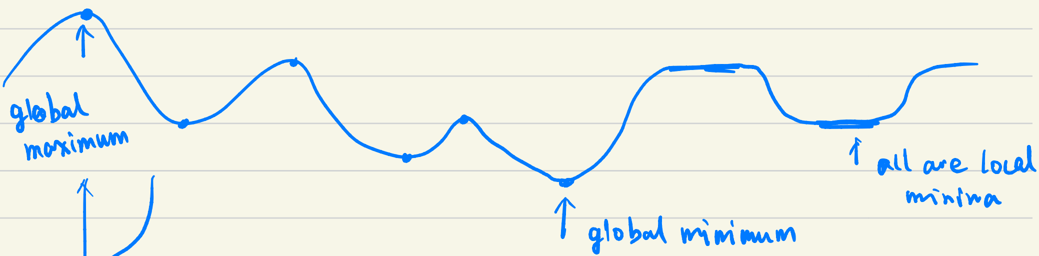
Fastest ascent direction

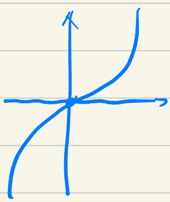
$\underset{\|u\|_2=1}{\arg\max}\ \nabla f(x)\cdot u = \dfrac{\nabla f(x)}{\|\nabla f(x)\|_2}$ (Cauchy - Schwarz)



$a \cdot b = \|a\|\,\|b\| \cos\theta$

Local/Global maxima/minima





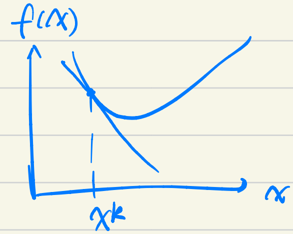
Line search methods

$f(x) \rightarrow \underbrace{f(x-\eta\nabla f(x))}_{g(\eta)}$

Exact line search

$\underset{\eta \geq 0}{\arg\min}\ f(x-\eta\nabla f(x))$

Back-tracking line search



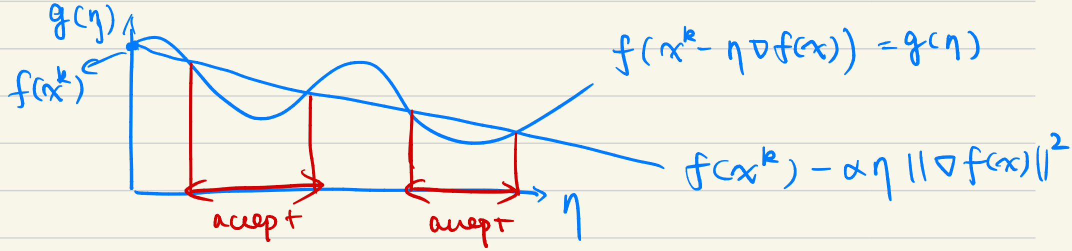
$$f(x^{k+1}) = f(x^k - \eta \nabla f(x))$$

$$\hat{f}(x^{k+1}) = f(x^k) - \eta \underbrace{\nabla^T f(x) \nabla f(x)}$$

$$= f(x^k) - \eta \|\nabla f(x)\|_2^2$$

Goal: Find η such that

$$f(x^{k+1}) \le f(x^k) - \alpha \eta \|\nabla f(x)\|_2^2$$



Start $\eta = 1$, set $\eta = \eta \cdot \beta$ (e.g. $\beta = 1/2$) until

$$f(x^{k+1}) \le f(x^k) - \alpha \eta \|\nabla f(x)\|^2$$

$$\alpha \in (0,1)$$